Michael A. Jones, Rabab Abi-Hanna, and Kenneth Krott

Applying Secant Lines and Tangent Lines for the Optimal Placement of Range Lights

Range lights are pairs of lighthouses on bays, rivers, and other waterways that guide boats safely along a linear path, called the *range line*. Because lighthouses are expensive to build, operate, and maintain, it is reasonable to minimize the number of lighthouses in use by ensuring that the range lines are as long as possible. The portion of a river that is deep enough for boat traffic is referred to as the river's channel. The channel is dredged often to ensure an appropriate depth for safe passage of boats. To keep a boat in a channel bounded by the curves $y = f(x)$ and $y = f(x) + k$, the optimally long range lines are the secant or tangent lines to the two curves. For example, a boat traveling from left to right on the river shown in **figure 1** should stay in the shaded channel.

Range lines are comparable to the lines made by connecting two consecutive channel markers that indicate the channel's limits or boundaries. For boats traveling upstream or from open waters, red buoys or red triangular signs mark the channel's left (port) boundary. Green buoys or green square signs mark the channel's right (starboard) boundary. For visibility at night, the markers often have flashing lights of the same color. To ensure that the water is deep enough for safe travel, the boat must remain in the channel between the sets of markers. Both range lights and channel markers use line segments to approximate curves. Range lights use segments to provide a path between the channel's boundaries, while channel markers use segments to provide approximations of the boundaries themselves.

Aerial views of the paths taken by runners on a marathon course or paths taken by race cars on a sinuous race course provide other examples of line segments that create paths between two boundary curves. The boundary curves define the courses and the paths between the curves that are acceptable routes. However, runners and race cars follow straight lines between the two boundary curves because straight lines minimize the distance traveled.

An application such as the range light activity can connect with students on many levels, from the practical to the theoretical. Practical applications often reinforce theoretical ideas in students' minds, emphasizing the utility of mathematics. Being able to apply basic mathematical concepts to real-life situations changes the way mathematics is taught in schools. Making problems interesting and meaningful to students is an everyday challenge. A compendium of diverse applications improves not only the quality of instruction but also the quality of learning.

This department is designed to provide in reproducible formats activities appropriate for students in grades 7–12. The material may be reproduced by classroom teachers for use in their own classes. Readers who have developed successful classroom activities are encouraged to submit manuscripts, in a format similar to "Activities" already published. Of particular interest are activities focusing on NCTM's curriculum standards, its expanded concept of basic skills, problem solving and applications, and the uses of calculators and computers. Send submissions to "Activities" by accessing **mt.msubmit.net**.

Another source of activities can be found in NCTM's *Using Activities from the "Mathematics Teacher" to Support Principles and Standards*, edited by Kimberley Girard and Margaret Aukshun (order number 12746; $35.95), which also includes a grid to help teachers choose the activities that best meet the needs of their students.

Edited by ***Ruth Dover***
dover@imsa.edu
Illinois Mathematics and Science Academy
Aurora, IL 60506

Dot Doyle
doyle.dot@gmail.com
North Carolina School for Science and Mathematics (retired)
Durham, NC 27715

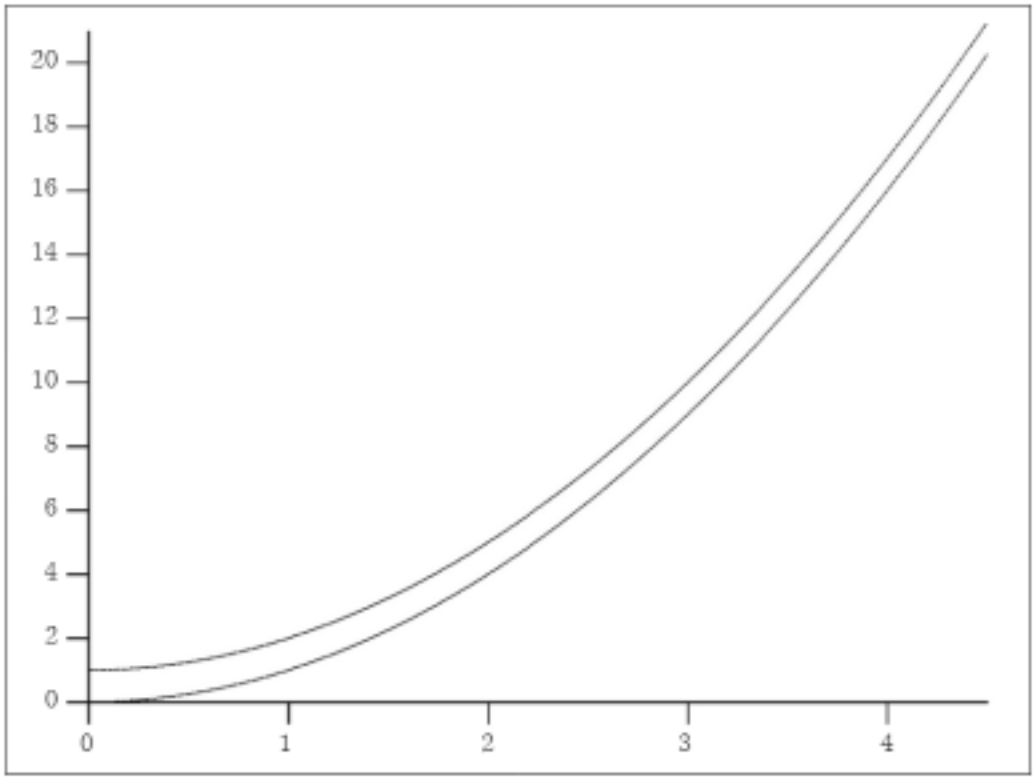

Fig. 1 River 1's channel

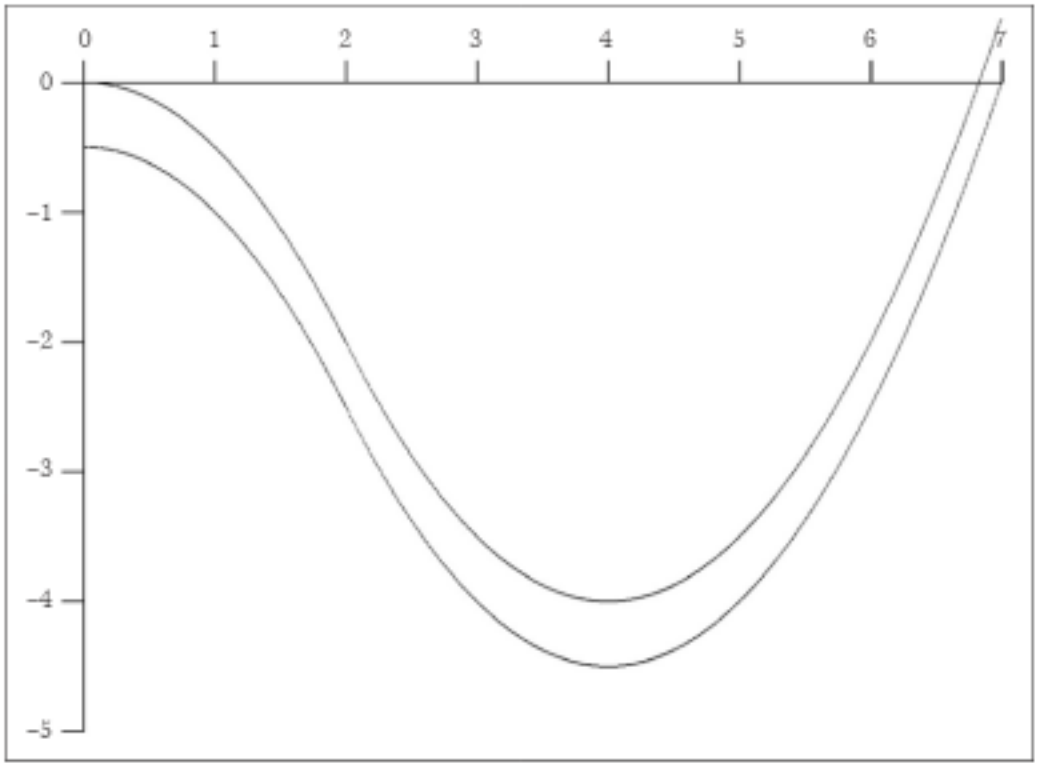

Fig. 2 River 2's channel

From the May 2008 issue of MATHEMATICS teacher